Teaching in a Different Direction

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Changes in teaching practice have not kept pace with the changes envisioned in numerous reform documents. In this paper, we present the results of a study of an experienced secondary teacher who taught a sequence of modelling tasks designed to investigate exponential growth and decay. Our results suggest that changes occurred in her practice along three dimensions: her role in listening and questioning, a new emphasis on student explanation and justification, and a more fluid view of the representations of the concepts.

Despite the large body of research on children's learning and a correspondingly large body of curricular materials that reflect this research, the goal of substantial change in teaching practices remains elusive. Many arguments are given for the reasons for this difficulty, including the challenges inherent in changing the larger systems within which the activity of teaching is embedded (Confrey, Castro-Filho & Wilhelm, 2000; Grant, Peterson & Shojgreen-Downer, 1996), the limits of teachers' mathematical knowledge for teaching (Cooney, 1999), and the lack of a professional knowledge base for teaching (Hiebert, Gallimore, & Stigler, 2002). We wish to argue that research on the development of teachers' knowledge in practice is an essential step to building the professional knowledge base needed to make progress in changing the practices of teaching. In this paper, we present the results of a study that examined the development of an experienced secondary teacher's professional knowledge as she implemented a unit on exponential growth and decay.

Theoretical Framework

Knowing how to teach means knowing how to see and interpret the events of classroom practice. Understanding teachers' knowledge means knowing how teachers interpret the practical problems of the classroom, how those interpretations evolve over time and across settings, and how and when those interpretations influence decisions and actions in the classroom. It is not enough to see what it is that teachers do in particular settings; we also need to understand how teachers interpret the particular events and how those interpretations are integrated with past experiences and influence subsequent interpretations. In other words, we are interested in understanding how teachers see and interpret their practice, and how these interpretations shift and change. A models and modelling perspective on teachers' knowledge development (Doerr & Lesh, in press) brings to the foreground a focus on the ways teachers interpret their practice. Teaching mathematics is much more about seeing and interpreting the tasks of teaching than it is about doing them. It is precisely a teacher's interpretations of a situation that influence when and why as well as what it is that the teacher does.

This research is grounded in the notion that the tasks of teaching can be seen as a complex and ill-structured knowledge domain and that expertise in such a domain requires the flexible use of cognitive structures to accommodate partial information, changing or unclear goals, multiple perspectives, and uncertain consequences (Feltovich, Spiro, & Coulson, 1997; Lampert, 2001). This perspective allows us to see that expertise in teaching is pluralistic, multidimensional, variable, contextualised and continual. Teacher education research has tended to capture this complexity and situated variability through an emphasis on teachers' beliefs or belief systems which serve as filters for teachers' knowledge and account for the variability in teachers' practices and for the resistance to changes in practice, but not for the often found disconnect between professed beliefs and actual practice. From a models and modelling perspective, the nature of teachers' knowledge is not a uniform, consistent or fixed set of constructs, but rather it's characterized as evolving along multiple dimensions. In this research project, we examined the evolving expertise of an experienced secondary teacher by focusing on how the teacher saw and interpreted the events in her mathematics class. We recognize both the complexity of those events and the extent to which the teacher's interpretations are grounded in the context and constraints of her practice (Borko, Mayfield, Marion, Flexer, & Cumbo, 1997; Leinhardt, 1990). At the same time, we conceptualise the teacher's knowledge as that of evolving or emerging expertise along multiple dimensions of practice. The central questions for this study were:

- How do teachers interpret the development of students' ways of reasoning about exponential functions?
- How do teachers' interpretations of students' development influence their actions in the classroom?

Methodology and Data Analysis

This study is part of a two-year research project using modelling tasks to investigate the teaching and learning of the mathematics of exponential growth and decay. Using the methodology of the multi-tiered teaching experiment (Lesh & Kelly, 1999), the development of students' reasoning is examined by teachers and the development of teachers' reasoning is examined by researchers. All three sets of participants were engaged in a cyclic process of interpreting tasks, and revising, refining and sharing their developing ideas about the tasks. The students were engaged in making sense of a sequence of model development tasks (Lesh, Cramer, Doerr, Post & Zawojewski, in press) that began with the well-known problem in which the number of pennies on each square of a checkerboard is double the number on the previous square, beginning with one penny on the first square. This was followed by five tasks: (1) generalizing the patterns of bacteria growth; (2) an exploration of the patterns of differences and ratios for linear, quadratic, and exponential growth; (3) the application of these patterns to population growth; (4) the extension to stochastic growth and decay; and (5) an exploration of transformations.

The teachers were engaged in making sense of the strategies that students might pursue when engaged with this sequence of tasks and of the pedagogical strategies that they might use in supporting the development of student learning. The researchers were engaged in understanding the teachers' interpretations of the student work and their subsequent teaching strategies. The teachers and researchers saw this sequence of model development tasks as one that would engage the students in a higher level of problem solving and would require some re-negotiation of classroom norms. The students in this study were in the 11th or 12th grade (16 - 18 years old) and studying pre-calculus. There were 6 boys and 7 girls in a class that was taught by a teacher with more than 30 years experience. The teacher had participated in two summer workshops where the problem sequence was the topic of teacher investigation. Just as the modelling activities were a new and non-traditional type of curriculum for students, so were the emerging pedagogical strategies new and non-traditional for the teacher. Hence, we interpret the teacher's implementation of this sequence as a model-eliciting event for her within her practice. It was intended that teaching these tasks would elicit new pedagogical models for her and extend her practice in new directions. Since the teacher participated in monthly meetings during the school year with colleagues and the researchers, she was able to share and re-use teaching strategies with the activities, thus refining how she interpreted the best ways to interact with her students and the modelling tasks. In our analysis, we conceptualised the teacher's knowledge as evolving along multiple dimensions of practice, including her view of her role in supporting student learning, her understandings of students' developing reasoning, and her mathematical understanding of the task.

The data sources included researchers' field notes from classroom observations of five lessons, the videotapes of the lessons, the transcriptions of the videotaped lessons and informal conversations with the teacher that occurred after the lessons, and semi-structured interviews before and after the lesson sequence. The analysis of the data took place in two stages. The first stage of analysis involved open-ended coding (Strauss & Corbin, 1998) of the field notes and the transcripts of each lesson. This was followed by viewing the videotapes for each lesson, and adding annotations and clarifications to the transcript that were visible from the videotape. Each author did this coding independently for the first lesson. We then met to compare our codes; differences in coding were resolved by finding references to early codes and making comparisons and revisions to the codes. The second stage of the analysis consisted of finding clusters of codes that defined the critical features or characteristics for each lesson. In keeping with our theoretical framework, we examined the teacher's reasoning about students' reasoning by focusing on how the teacher saw and interpreted the events in her class. The dominant events that governed each lesson were then examined for themes that cut across the lessons and reflected shifts in the teacher's perspective and interpretations. These themes are the organizing framework for our results.

Results

The teacher interpreted the sequence of modelling tasks as reform-based or non-traditional curricula. During the interview prior to implementing the modelling activities, the teacher acknowledged these activities required her to teach in a "different direction". A thirty year veteran, the teacher interpreted a significant part of her role as that of presenting and explaining to her students the processes and concepts of mathematics. Changing her role from that of presenter and explainer meant that she would need to do more listening and less telling. She knew she would be engaging with students in newly evolving ways. This also required her to think about mathematics in terms of the ways students communicated their ideas. Our findings about the teacher's interpretations of the learning activities are organised around the ways in which we see that she shifted her perspectives from a more traditional practice:

- about her role in explaining and presenting information
- about her students' roles as re-tellers of the information
- about the nature of the mathematical representations of growth and decay

The teacher in this study had many years of experience in teaching the topics of pre-calculus. While she acknowledged the need for her students to engage in higher level problem solving and more mathematical reasoning, she was both cautious and skeptical about implementing alternative curricular tasks. She knew that her students needed skills in algebra to succeed in subsequent courses in mathematics, and she also was concerned that they not get discouraged in their study of mathematics. The teacher had spent time carefully planning which of the modelling tasks to use to best prepare her students for calculus, their following course. She saw that there was mathematical power in revisiting the concept of rate throughout the tasks and in engaging the students in developing generalised forms for exponential growth and decay functions. Even though the tasks spiralled around these powerful concepts, the teacher thought that truly convincing evidence of student understanding and retention would only be visible when they took subsequent courses. As such, the teacher was reluctant to view the modelling tasks as replacements for the traditional pre-calculus curriculum.

Moving From Teaching as Telling to Teaching as Listening and Questioning

Much of the time during the activities in the lesson sequence, the students discussed the task and worked out the analysis in small groups. The teacher moved among the groups, listening to the students' discussions. One problem in the first lesson of the sequence required students to investigate the patterns of bacteria growth for a culture that had an initial size of 1500 bacteria and doubled every half-hour. While students worked to create a table, graph and equation for this scenario, the teacher observed their efforts. For example, in interacting with one group of students, she listened for and encouraged student ideas as well as focused the task as they tried to determine the general equation while studying the table they generated for this problem:

T:	You now have to find the equation that will mimic that pattern.
S1:	So it's not 2^(x-1)?
T:	You have to study the pattern that's there.
S2:	Like 200*2
T:	You've got an idea, now work with it. (Lesson 1, Field notes, p.5)

The teacher focused the students' attention on the use of patterning and encouraged them to work with their idea. Later, with another group, a student described his partial solution. The teacher redirected his thinking to a comparison with the thoughts of others in his group and re-focused his attention to the central question of the task:

> T: OK, but compare your equation to others in the group. We need to see a relationship between the time and the size of the colony. (Lesson 1, Field notes, p.6)

As she listened to this student's ideas, she refrained from telling him the next step in solving the problem. The teacher's emerging model of teaching included reminding students of the goal of the task and redirecting their thinking.

Another dimension of the teacher's shifting practice included strategies for moving the class from small-group work to a whole class discussion of the problem. As she circulated among groups, she would decide to ask certain group members to write their work on the chalkboard. We cannot be certain of the reasons that she chose particular groups' work and not others; however, we did notice that she selected a variety of solutions for class discussion. This sharing of multiple solutions to the bacteria problem required the teacher to understand student thinking about the mathematics of the task and to weigh the value of discussing various solutions. She had to listen to and process the accuracy and meaning of each groups' equation as they developed it from their table. The teacher had the class look at two particular solutions to a problem in the sequence where the bacteria of a culture doubled every 15 minutes and had 80000 bacteria present after 1 hour and 15 minutes. This allowed her to question the class about how their solutions were different and why. The teacher asked Jack and Jessica to explain their charts and equations (see Figure 1).

Time(x)	Bacteria(y)
0	2500
1	5000
2	10000
3	20000
4	40000
5	80000

Time(x)	Bacteria(y)
0	2500
15	5000
30	10000
45	20000
60	40000
75	80000

Figure 1. Jack's chart and Jessica's chart.

T: Jack, would you explain how you did this?
Jack: I started at 80000 and went backwards. So every 15 minutes it doubles, so every 15 minutes I took half of 80000.
T: So what does the domain represent?
Jack: Every time it doubles.
T: And what's different about your chart, Jessica?
Jessica: I just wrote out the minutes.
T: And both of these would make sense. You could think of it as a block of time or the actual minutes. So to find an equation, you made a comparison of the columns and what did you get?
Jessica: y = 2500*2^(x/15)
T: Can you explain how you got that?
Jessica: We used trial and error in the calculator, used 15 for the time until it matched.
T: So what does x stand for?
Jessica: Hm.
T: What unit of time are you considering?
Jessica: I'd say minutes.
T: So what's the 2500 represent?
Jessica: The starting number.
T: And where does the 2 come from?
Jessica: The bacteria are doubling.
T: And Jack has 0 through 5 in his table, so would the same equation work?
Jack: No, it would be y=2500*2^x
T: So even though we have one situation yet two equations to represent it, which we use depends on what x stands for. (Lesson 2, fieldnotes, p.2-4)

In focusing on these two solutions, the teacher questioned the students to clarify her understanding of the students' understanding about the differences between the equations. She summarised that using different time increments necessarily changed the equation, and she was able to validate student thinking for both solutions. We see this as a shift in role from the teacher as explainer and presenter to listener and questioner. Her evolving practice included summarising students' thinking rather than evaluating whether or not they had recited back concepts she would have otherwise told them about exponential growth functions. We infer that this shift in her interpretation of student thinking happened because her implementation of modelling activities.

Moving From Student Learning as Re-telling to Student Learning as Explanation and Justification

In listening to student thinking during the modelling activities, we found that the teacher made some shifts in her perspective on what she expected of her students. She asked the students to explain their solution processes and in some cases to elaborate their partial solutions. As students gave these explanations, we found that the teacher expected students to justify why they were thinking in certain ways. As noted in the above example from the whole class discussion of Jack and Jessica's tables, the students had to reason through why the equations would be different based on a different unit for the independent variable (Lesson 2, transcript, lines 70-80). We see the teacher's emerging pedagogical model included finding evidence of student learning as they explained their reasoning.

During a whole class discussion of the population growth model, the teacher asked a student to justify why she rounded certain values. The growth model concerned the population of a town that was 1500 in 1860 and grew at a rate of 2.8% per year until 1960. Students were asked to determine the population in 1940. It was expected that students would apply their earlier strategies for analysing a table of values and investigating the patterns in the entries. The teacher expected clear reasoning for representing real-world population values as she questioned Kerry about her table values:

> Kerry: I multiplied the initial [population] by 1.028 each year.
> T: And then you got some decimals. What did you decide to do with the decimals?
> Kerry: I rounded up on some and then rounded down on some.
> T: How did you decide?
> Kerry: If it was like .7, maybe they were just missing an arm. I counted them as a whole person (laughing). If it was .2, I said it was probably just a foot and rounded it down.
> Andra: Wouldn't you always round down?
> T: Why would you think that?
> Andra: Because .7 isn't a whole person.
> T: Your arguments are both valid and this is a problem in mathematics where we can have different opinions. What is going to happen is Andra, you're going to get a few less people and Kathleen's going to get a few more, so it's not going to make a whole lot of difference here. (Lesson 3, transcript, p. 14)

Students' varying interpretations of how to make sense of real-world scenarios served as a catalyst to shift the teacher's perspective about what constituted student learning. She not only expected student learning to include logical reasoning and proof of their own thinking but also understanding the thinking of others. We posit that her emerging pedagogical model included encouraging and supporting students' thinking for themselves rather than telling them how or what to think when solving a particular problem.

Moving From Mathematics as a Static Set of Rules to Mathematics as a Fluid Set of Multiple Representations of Generalizations

The sequence of exponential modelling activities that the teacher implemented offered her a fresh perspective on the multiple ways student ideas might develop. As we noted in the above example, students made mathematical decisions that produced differing representations of the same concept. In contrast to a more traditional view that would emphasize one preferred way to do and represent mathematical ideas, we contend that the modelling activities were a catalyst for the teacher to shift her perspective about the range and fluidity of mathematical representations. The teacher made pedagogical choices that served to bring forth discussion of multiple representations of exponential concepts. This was illustrated by her treatment of student solutions to the population growth task described above. To facilitate the whole class discussion of the problem, the teacher listened to small-group discussions and chose two students from the same group to present their work. She asked Kerry to put her chart on the board and Imad to write his generalised form of a growth equation for the data (see Figure 2).

x (year)	y (population)
1860	1500
1861	1542(=1500*1.028)
1862	1585(=1500*1.028*1.028)
1863	1628
1864	1675
1865	1722

$$y = (\text{base}\ \#) \cdot (\text{rate of increase})^{(x/x\ \text{increments})}$$
$$y = 1500 \cdot 1.028^{x}$$

Figure 2. Kerry's chart and Imad's equations.

Our interpretation of the teacher's choice is that she analysed in-action the connection between students' representations and used the connection between student representations as a means to further the discussion of the underlying ideas of exponential growth. The teacher asked each student to explain:

> T: Kerry, how did you get the 1542?
> Kerry: 1500 times 1.028 (T goes to board and writes this pattern next to the entry in Kerry's table)
> T: And how did we get the 1585? It was the 1542 and we would do it again (writing out multiplication by 1.028 again). Now that is going to lead us into…Imad, go up and tell them about your formula up there.
> Imad: I was working this out last class or something so I don't remember why so I don't think I can explain it too well right now. But this says y equals the base number of the y value, which is 1500 in this case. Times the rate of increase-however much it's increasing as you go down the y value. Raised to the x over the increments on the x value, which would be in this case one year each. So it says 1500 times 1.028 which is 2.8%. If you change it into a normal number it's .028 but then you have to add it which is a pain, so you can just do 1.028 and that's the rate of increase. And then you just raise it to the x over the time increments which is just one year so it's just x over 1. And that's the formula.
> T: It's a good formula, we're going to work with it but it's not a final product.
> (Lesson 3, Fieldnotes, p. 9).

Even though the teacher did not see Imad's generalised equation as a final product, she affirmed that it was an idea that they would work with. The fact that she chose these interconnected student representations indicated to us that she was aware of students' thinking and looked for various mathematical representations of the same concept within student work. The validation of partial solutions and use of different representations of mathematical concepts indicate to us that the teacher's pedagogical model included a widening perspective on what is mathematics and that student's emerging representations are fluid and dynamic rather than static.

Discussion and Conclusion

Even though the teacher was cautious in her implementation of the new materials, the sequence of modelling tasks was a site for the development of student learning and a site for the development of the teacher's interpretations of the tasks of teaching and learning. In keeping with our theoretical framework, we found the teacher's practice evolved along three dimensions: her role in explaining and presenting; her students' roles in explaining and justifying; and her view of the nature of useful mathematical representations for teaching and learning. The teacher in this study shifted from a role as the primary explainer and presenter in the classroom to a new role as listener and questioner. The teacher began to engage the students in explaining and justifying their reasoning and their representations. At the same time, she maintained her role in summarising students' thinking to conclude class discussion. These actions appeared to increase the level of expectations that she had for her students, beyond following algebraic procedures. We found that the teacher carefully chose student solutions to provide opportunities for the students to defend their reasoning and she was able to make connections between alternative student representations. In this way, her view of the range of acceptable mathematical solutions appeared to widen and become more fluid in terms of the particulars of her students' thinking. Within the constraints of her practice, the teacher made some changes toward teaching in a different direction. These changes appear to have been a consequence of engaging with students in learning through tasks that required her to listen to and respond to their ways of reasoning and representing their ideas.

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